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On Sun-spot Periodicities.—Preliminary Notice.

By ARTHUR SCHUSTER, F.R.S.

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In cases where it is necessary to separate true periodic changes from other variations, which during short periods of time often simulate periodicities, the method of the periodogram is at present the only one which can give definite results. In view of the importance of the questions connected with the changes in the frequency of the sun-spots, I have, therefore, undertaken the considerable labour of forming a complete periodogram of sun-spot variability as far as the data at my disposal allow me to do so. The following is a brief abstract of the results; the investigation will be presented shortly:—

The periodogram, as already explained, is the diagram representing the intensity of periodic variations as determined from the sum of the squares of the two Fourier coefficients belonging to each assumed period. This diagram represents for any regular or irregular change exactly what the energy diagram gives us for a luminous disturbance which is analysed by a spectroscope.

Periods of not more than three or four years' duration may be satisfactorily studied by means of the tables of sun-spot areas which are at our disposal. For periodic times which exceed four or five rotations of the sun, I have used the mean daily areas in each synodic rotation of the sun as collected and published by the Solar Physics Committee. The data reach back as far as 1832. For shorter periods we have to go back to the Greenwich measurements, which are published for each day since 1883.

In the investigations of periods longer than three years, I have used Wolf and Wolfer's sun-spot numbers, which are published for each month beginning with the year 1749, and based on a combination of spot and group counting.

The periodogram of the whole interval of 150 years showed, as was expected, a marked peak corresponding to the 11-year period, but it presented some features which rendered it desirable to investigate separately each of the two intervals of 75 years into which the whole range of time may be divided.

The result was surprising, for the two separate periodograms seem at first sight to have nothing in common. From 1750 to about 1825 the observations do not indicate any marked periodicity of approximately 11 years,

but two periods of $13\frac{3}{4}$ and $9\frac{1}{4}$ years respectively. Since that time, on the contrary, the variations are well represented by an almost homogeneous period of 11.12 years. The apparent absence of the 11-year period in the last half of the eighteenth century cannot, in my opinion, be attributed to the unreliability of the observational data.

The method employed is specially suited to investigate possible effects of planetary configurations, which have often been supposed to exist. I have therefore specially investigated the rotation period of Jupiter and the synodic periods of revolution of the planets Venus and Mercury. My results do not show any trace of a planetary influence.

The most persistent period which I have found is one of 4.81 years' duration. It appears, with good agreement of phase, in Wolf's sun-spot numbers both before and after 1825. It also shows well in the direct measurements of sun-spot areas. The amplitude of this period seems to be about one-sixth of that of the main 11 years' periodicity.

An increased intensity is observed for periodic times of 5.625, 3.78, and 2.69 years, the importance of the periods being in descending order. Multiplying these numbers by 2, 3, 4 respectively we obtain 10.76, 11.34, 11.25. These periodicities are, therefore, in all probability only sub-periods of the 11 years' variation.

The only remaining periodicity which is clearly indicated by the periodogram is one of 4.38 years' duration, but this does not occur in the observations previous to 1750, and only shows feebly in Wolf's series of figures as drawn from the observations of the last seventy years. Its real existence is, therefore, doubtful, though some prominent outbreaks during the last fifteen years may be associated with it.

Special care was devoted to the investigation of periodicities in the neighbourhood of 26 and 27 days. Some rise in the periodogram for periods approximating to the time of synodic solar rotation may be expected if an appreciable number of sun-spots live through several rotations. But apart from the fact that different circles of latitude have different periods of revolution, only a very broad band (in the optical sense) can be expected, as the number of rotations during which a sun-spot is observed very seldom exceeds three. Such a succession could only produce a general rise between the periods of 25 and 29 days; this rise does, indeed, seem to take place, but no definite periodicity has been discovered.

[*Note added December 12.*—Since writing the above, I have found strong evidence of the reality of a cycle having a periodic time of about 8.37 years. This evidence, which seems to me convincing, is briefly as follows:—If by

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means of the table published by the Solar Physics Committee, giving the mean daily areas of spots for each year since 1832, the average value for the eleven-year cycle in each year is calculated and deducted from each entry, we obtain a series of numbers which may be taken to represent the sun-spot variation with the eleven-year period eliminated. This series shows decided maxima, which took place in the years 1836, 1845, 1853, 1862, and 1870, the intervals being alternately nine and eight years. The periodogram based on Wolf's numbers for the total interval 1749 to 1900 shows a decided maximum for a periodicity of 8·25 years. Provisionally accepting this period and taking the phase as obtained from the interval 1749 to 1826, we may forecast the maxima for the subsequent interval. We thus obtain: 1836·3; 1844·7; 1852·9; 1861·2; 1869·4 in almost exact agreement with the above. Taking into account the apparent shift of phase which is noticed when the observations of the first and second portions of the complete interval are taken into account, the more exact time of the period seems to be 8·38.

A further confirmation of this period may be obtained by means of an empirical criterion which I have found useful. In a true periodicity, the main sine or cosine variation is in the spot cycles accompanied by the higher harmonic which has a period of half the length of the main period. This harmonic is distinguished by a rise in the periodograph curve. Such a rise where it exists gives increased probability to the real existence of the period considered. In the case of fictitious periods, on the other hand, I have generally found that when the intensity of a certain period is exceptionally great, the whole of the accidental variations being as it were concentrated into it, the semi-period shows an exceptionally low intensity. As regards the cycle under consideration, it is therefore significant that the periodogram has a maximum for the period of 4·125 years in both the intervals 1750 to 1826 and 1826 to 1900.

In tracing backwards this new period as well as the one which is generally admitted to have a periodic time of a little over 11 years, the conclusion has been forced upon me that while the times of the maxima seem to occur with almost astronomical accuracy, the intensity of action is subject to great variations. This conclusion I believe to give the key to the explanation of the great irregularities which are observed in the succession of sun-spot cycles. While in the period 1750 to 1826 the eleven-year period does not appreciably affect the periodogram, it was not totally absent, for what there remains of it fits in well with the phase of the great eleven-year cycle which has been observed since 1826. I have approached this question without preconceived opinions, and my first impression on looking at the periodogram tended towards a denial of fixed periods extending over long ranges of time,

but gradually the conviction has forced itself upon me that there are a number of perfectly definite periodicities, having the peculiarity that for a certain number of cycles they are effective and then cease to be active. Their real existence is proved by the fact that whenever they reappear after a period of inactivity, they form, without change of phase, a continuation of the former periodicities. The phenomenon reminds one to some extent of the beats of sound, but the evidence at present is against an explanation founded on the theory of interference of two nearly equal oscillations.

The periodogram for the total interval 1825 to 1900 shows a rise for a periodicity of about 13·5 years. Looking at the records of sun-spot maxima since 1600, we find three examples of two successive maxima being separated by an interval of about 13·5 years. They are: 1626·0 to 1639·5; 1816·4 to 1829·9; and 1870·6 to 1883·9. Taking the interval between 1639·5 and 1816·4, we find it to be 13 times 13·61, while the interval between 1829·9 and 1870·6 is three times 13·57. This would tend to show that the maxima in question belong to a periodicity of about 13·57 years, which has extended right through two and a-half centuries. It is specially to be noted that no observation previous to 1749 was used in determining the periodogram, so that the coincidence of the maxima as far back as 1626 is remarkable. Without wishing to attach too great a weight to what may quite possibly be only a numerical accident, I think that as a confirmation of conclusions arrived at independently, the coincidence deserves notice.

The periodicities which so far have been traced with a considerable amount of probability were found to have periodic times of 4·81, 8·38, and 11·125 years. There are indications that the first of these periods is somewhat less than the number given, and at one time I had put it down as low as 4·75, but the above were actually the times directly determined, and I adopt them so as to avoid suspicion of having in any way helped by personal bias to improve the following remarkable relationship between the numbers. That there was a connection between the numbers was only discovered after the first draft of my paper had been written out giving the above figures as highly probable periodicities. Taking frequencies into consideration in place of periodic times, we are led to consider reciprocals, and thus find—

$$\begin{aligned}(11\cdot125)^{-1} &= 0\cdot08989 \\ (8\cdot38)^{-1} &= 0\cdot11933 \\ \text{Sum} &= (4\cdot78)^{-1} = 0\cdot20922\end{aligned}$$

Hence the sum of the frequencies of two of the periods agrees, within the possible errors, with the frequency of the third period.

But it is also found that the two first numbers are very nearly in the ratio

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of three to four, so that we may also express the three periodic times as sub-periods of 33·375 years ; thus—

$$\frac{1}{3} \times 33\cdot375 = 11\cdot125$$

$$\frac{1}{4} \times 33\cdot375 = 8\cdot344$$

$$\frac{1}{7} \times 33\cdot375 = 4\cdot768$$

How far this connection is accurate or approximate, it is impossible to say at present, but the fact that the three periods which have been traced with a considerable degree of certainty should also bear a remarkably simple relationship to each other is worthy of note. If we accept a period twice as long as that given above, we might account for other periodicities of which at present the times are only approximately determined ; thus $\frac{1}{5} \times 66\cdot75$ would lead us to 13·34, in fair agreement with the period of 13·57 years which has been mentioned above. But for the present I do not wish to lay any stress on this.

I have confined the discussion to the statistical problem, but could not help giving some thought to the possibilities of explanation. It is not difficult to form a theory which should account for the peculiarity of interruption in the succession of cycles which has been found to be characteristic of the sun-spot periods. A reason may also be found for the first relationship between the cycles. But the harmonic dependence on a long period is more difficult to account for.]
